Descriptive Statistics

Standardizing Data

Z–Score

What is **Standardizing Data**?

Standardizing data is the process of putting different variables on the same scale. This process allows you to compare values from different samples such as exam results from different exams.

What does **Standardization** do?

Standardizing Data

▶ produces the number of standard deviations above or below the mean that a specific observation falls, and

▶ identifies the usual and unusual data element in the process.

What is **Z–Score**?

It is a numerical value, associated with data element once the **Standardization** of that data element is completed.

It tells us **how many standard deviations** the data element is **below or above** the **mean** .

How do we find the **Z–Score**?

It can be computed by the formula $Z = \dfrac{x - \bar{x}}{S}$.

Always round to three decimal places.

What is x in the **Z–Score** formula?

It represents the data element that we want to **Standardize**.

What is $\bar{x}$ in the **Z–Score** formula?

It represents the **mean** of the **sample**.

What is s in the **Z–Score** formula?

It represents the **standard deviation** of the **sample**.

Example:

Class exam had an average 78 with standard deviation of 6.8.

▶ Find the Z score for exam result 90.

▶ Find the data element associated with the Z score 2.5.

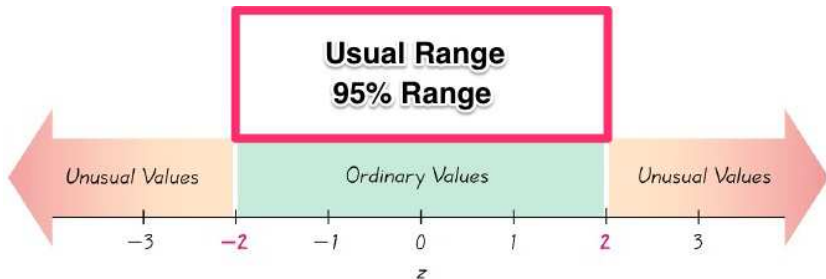Solution:

▶ For the Z score $\Rightarrow$ we use the formula,

$$Z = \frac{x - \bar{x}}{S} = \frac{90 - 78}{6.8} = \frac{12}{6.8} \approx 1.765$$

▶ $Z = \frac{x - \bar{x}}{S} \Rightarrow 2.5 = \frac{x - 78}{6.8} \Rightarrow 2.5 \cdot 6.8 = x - 78 \Rightarrow x = 95$

What are **Unusual** and **Ordinary** values?

Any data value that its **Z score** falls within -2 and 2 is considered an **ordinary** or **Usual** value.

The chart below clearly shows how to identify **Ordinary** and **Unusual** values.

Example:

John makes a monthly salary of \$5750 as a nurse at the local
hospital. The average salary for 25 randomly selected nurses was
\$5275 with standard deviation of \$225. Find

- ▶ Find the usual range of salaries according to the Z Score.
- ▶ Find the Z–score for John's salary.
- ▶ Is John's salary considered to be ordinary or unusual?

Solution:

- The usual range $\Rightarrow 5275 \pm 2(225) \Rightarrow 4825$ to 5725.
- Z–score $\Rightarrow Z = \dfrac{x - \bar{x}}{S} = \dfrac{5750 - 5275}{225} = \dfrac{475}{225} \approx 2.111$
- Ordinary or unusual? $\Rightarrow$ Unusual

Example:

Maria made 91 on exam 1 and 87 on exam 2 in her statistic class.
Below is the summary of exam results for both exams.

Exam 1	Exam 2
$\bar{x} = 85$	$\bar{x} = 76.8$
$s = 4.8$	$s = 6.8$

▶ Was any of her exam results unusual?

▶ What exam did she do better?

Solution:

▶ For unusual exam result, we first find the Z score for each exam.

Z–score (Exam 1)$\Rightarrow Z = \dfrac{x - \bar{x}}{S} = \dfrac{91 - 85}{4.8} = \dfrac{6}{4.8} = 1.25$

Z–score (Exam 2)$\Rightarrow Z = \dfrac{x - \bar{x}}{S} = \dfrac{87 - 76.8}{6.8} = \dfrac{10.2}{6.8} = 1.5$

Neither exam results are considered to be unusual.

▶ Z–score for exam 2 is greater than the Z–score for exam 1, therefore she performed better in exam 2.